be essential for the solution of Eq. (21) to be asymptotic since $\gamma_2 = \pm 90$ deg (zoom climb or dive) when $\lambda_{x_0} = 0$.[5] Whereas, in this case, $\gamma_2 \to 0$ as $(h_1^c - h) \to 0$, even for $k=0$. Figure 1 illustrates the dependence of γ_2 on $(h_1^c - h)$ and k for the F-8 aircraft.

Expanding the dynamics in Eqs. (28) and (29), we obtain

$$\mathrm{d}\delta h/\mathrm{d}\tau_2 = V_1\gamma \tag{42}$$

$$\mathrm{d}\gamma/\mathrm{d}\tau_2 = \delta L/mV \tag{43}$$

It can be shown that δL is related to γ and h by

$$\delta L = -K_3\gamma - K_4\delta h \tag{44}$$

where

$$K_3 = K_5\sqrt{\partial^2 H_2/\partial\gamma^2}, \qquad K_4 = K_5\sqrt{\partial^2 H_2^2/\partial h^2} \tag{45}$$

$$K_5 = \sqrt{-qsW/2\lambda_{E_1} KV_1} \tag{46}$$

Thus, the eigenvalues of the closed-loop system are given by the roots of

$$s^2 + K_3 s/mV_1 + K_4/m = 0 \tag{47}$$

It can be shown that the weighting parameter k enters into K_3 but not K_4. Thus, it affects the damping ratio but not the natural frequency in Eq. (47). Also note that k enters the control solution through Eq. (23).

Numerical Results

Aerodynamic and propulsion data for an F-8 aircraft was used to calculate and compare the eigenvalues from Eqs. (39) and (47). The calculation was performed at five energy levels along the climb path. For this aircraft, the long-range cruise energy is $E_0 = 19{,}100$ m. The first three columns in Table 1 compare the eigenvalues for $k=0$ (minimum time). Note that in every case the damping ration obtained using Eq. (47) is approximately half that obtained from Eq. (39). What is not apparent from Table 1 is that the natural frequencies are equal.

The weighting parameter k only affects the damping ratio and not the natural frequency of the closed-loop dynamics corresponding to Eq. (47). Thus, it is possible to tune the solution for a single energy level. For example, for $E = 9112$ m and $k = 1.78$, the eigenvalues from Eqs. (39) and (47) are equal. The calculation at other energy levels is summarized in the last column of Table 1. Note that a reasonably good approximation to the eigenvalues of Eq. (39) is obtained. A better approximation results when k is chosen for $E = 13{,}528$ m. In any case, the high degree of coupling that exists between h and γ dynamics is evidenced by the fact that the eigenvalues of Table 1 occur in complex conjugate pairs at all energy levels. The improved damping that results for $k>0$ has been verified through nonlinear simulation of the F-8 dynamics.[4]

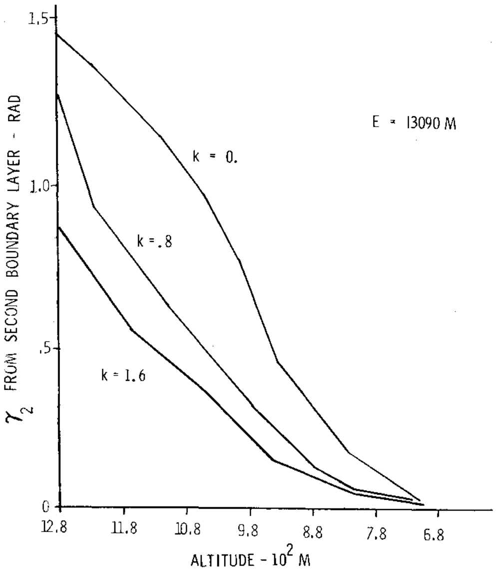

Fig. 1 Optimum flight-path angle from third boundary-layer solution.

Summary

This Note proposes a method for optimizing h and γ dynamics in a form suitable for on-line computation. It is shown that formal separation and boundary solution for these dynamics results in a control solution with insufficient damping. The eigenvalues of the linearized closed-loop dynamics along the optimal climb path can be made to closely approximate the linearized dynamics of the exact solution by introducing a penalty term on γ in the performance index.

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References

[1] Calise, A.J., "Singular Perturbation Techniques for On-Line Optimal Flight Path Control," *Journal of Guidance and Control,* Vol. 4, No. 4, July-Aug. 1981, pp. 389-405.

[2] Ardema, M.D., "Solution of the Minimum Time-To-Climb Problem by Matched Asymptotic Expansions," *AIAA Journal,* Vol. 14, July 1976, pp. 843-850.

[3] Ardema, M.D., "Linearization of the Boundary Layer Equations of the Minimum Time-To-Climb Problem," *Journal of Guidance and Control,* Sept.-Oct. 1979, pp. 434-436.

[4] Calise, A.J., Moerder, D.D., "Singular Perturbation Techniques for Real Time Aircraft Trajectory Optimazation and Control," NASA Contractor Report 3597, Aug. 1982.

[5] Calise, A.J., "Extended Energy Management Methods for Flight Performance Optimization," *AIAA Journal,* Vol. 15, March 1977, pp. 314-321.

A Flexible Structure Controller Design Method Using Mode Residualization and Output Feedback

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Introduction

PROPOSED large flexible space systems (LFSS) will involve multiple actuators and sensors, providing the prospect of integrated control algorithms for attitude control and structural mode damping. The analysis and synthesis of

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such controllers requires large-scale computer programs, both to implement multivariable control theory calculations[1] and to manage the large amount of data associated with dynamic models of such systems. A widely used approach for multivariable control design is linear-quadratic-Gaussian (LQG) theory. However, this theory does not accommodate directly the effects of errors due to truncating the order of the dynamic model, and reduced-order compensator designed by LQG methods can induce instability through "spillover."[2]

This paper reports on a study of mode residualization[3,4] for reducing the order of compensators obtained by LQG design, and an extension of an output feedback technique,[5] developed to alter the compensator in order to overcome instability induced by the residualization. While the technique does not guarantee stability, it has shown some success and offers an approach to reduced-order compensator design that requires no greater sophistication in control theory than that necessary to implement LQG design methods.

Derivation

In typical state space notation, a linear dynamic system can be represented by

$$\dot{x}=Ax+Bu, \qquad z=Cx \tag{1}$$

where x is the n-dimensional state vector, u the control vector, z the observation vector, and A, B, and C constant matrices of appropriate dimensions. LQG theory provides a capability to design constant gain matrices F and G, which implement an estimator and regulator of the form

$$\dot{\hat{x}}=A\hat{x}+Bu+G(z-C\hat{x}), \qquad u=-F\hat{x} \tag{2}$$

where $\hat{x}$ is the estimate of x.

These equations represent an nth order compensator, which in itself is a linear dynamic system and can be transformed to modal (or block diagonal) form by a coordinate transformation, $\hat{x}=T\hat{\xi}$, so that the compensator becomes

$$\dot{\hat{\xi}}=\Lambda\hat{\xi}+\Gamma z, \qquad u=-H\hat{\xi} \tag{3}$$

where, by definition,

$$\Lambda=T^{-1}(A-BF-GC)T$$

$$\Gamma=T^{-1}G, \qquad H=FT$$

One method for reducing the order of the modal compensator is to apply the formalism of mode residualization.[3,4] This requires partitioning of Eq. (3)

$$\begin{bmatrix}\dot{\hat{\xi}}_c\\ \dot{\hat{\xi}}_r\end{bmatrix}=\begin{bmatrix}\Lambda_c & 0\\ 0 & \Lambda_r\end{bmatrix}\begin{bmatrix}\hat{\xi}_c\\ \hat{\xi}_r\end{bmatrix}+\begin{bmatrix}\Gamma_c\\ \Gamma_r\end{bmatrix}z$$

$$u=-(H_cH_r)\begin{bmatrix}\hat{\xi}_c\\ \hat{\xi}_r\end{bmatrix} \tag{4}$$

where $\hat{\xi}_c$ is the p-dimensional modal state vector of the desired pth order compensator, and $\hat{\xi}_r$ is the $(n-p)$-dimensional modal state vector of the residualized modes. These latter modes are treated as though they were continuously in steady state, so that

$$\dot{\hat{\xi}}_r=0 \qquad \text{and} \qquad \hat{\xi}_r=-\Lambda_r^{-1}\Gamma_r z$$

Thus, substitution gives the reduced-order compensator

$$\dot{\hat{\xi}}_c=\Lambda_c\hat{\xi}_c+\Gamma_c z, \qquad u=-H_c\hat{\xi}_c+U_0 \tag{5}$$

where $U_0=-H_z z$ and $H_z=-H_r\Lambda_r^{-1}\Gamma_r$.

When this technique is applied to a full-order compensator, which has been designed to improve the damping on a structural model with closely spaced modes, it can induce instability in a few modes at frequencies slightly higher than those retained in the reduced-order compensator. One approach to restabilizing these modes is to consider the system of Eqs. (1) and (5) as a dynamic system being controlled by output feedback matrix H_z; that is,

$$\begin{bmatrix}\dot{x}\\ \dot{\hat{\xi}}_c\end{bmatrix}=\begin{bmatrix}A & -BH_c\\ \Gamma_c C & \Lambda_c\end{bmatrix}\begin{bmatrix}x\\ \hat{\xi}_c\end{bmatrix}+\begin{bmatrix}B\\ 0\end{bmatrix}U_0$$

$$z=(C\;0)\begin{bmatrix}x\\ \hat{\xi}_c\end{bmatrix} \tag{6}$$

The question then arises whether an H_z can be found to stabilize this system.

The present study has investigated obtaining H_z by an extension of the output feedback method of Patel.[5] This method first designs a *state* feedback gain F' to stabilize the system of Eq. (6), then an *output* feedback gain H_z is sought to produce the same closed-loop pole locations, which requires that

$$U_0=-F'\binom{x}{\hat{\xi}} \quad \text{and} \quad F'=[H_zC\;0]$$

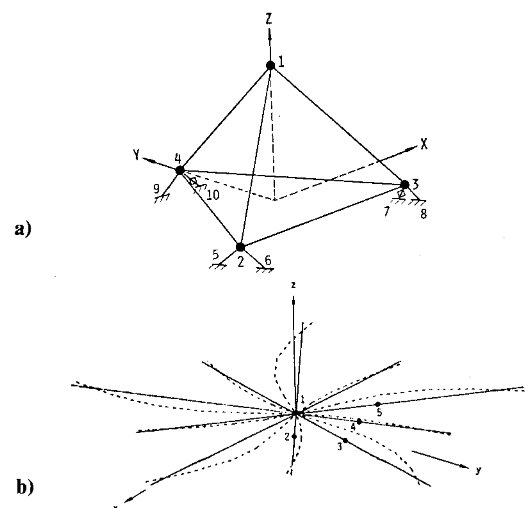

Fig. 1 Tetrahedron and 10 boom space structure models.

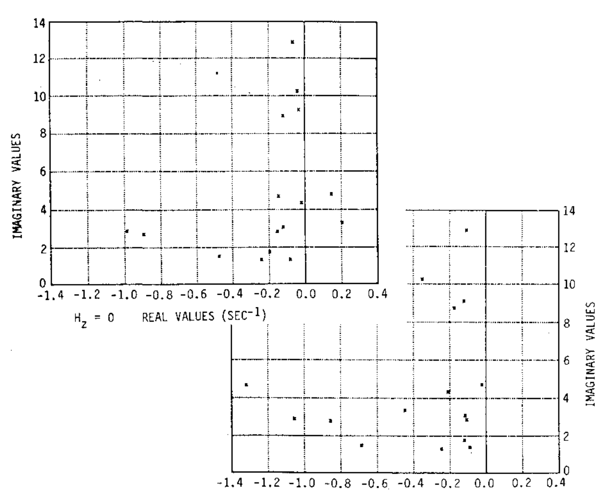

Fig. 2 Mode residualization and output feedback, 12-mode tetra.

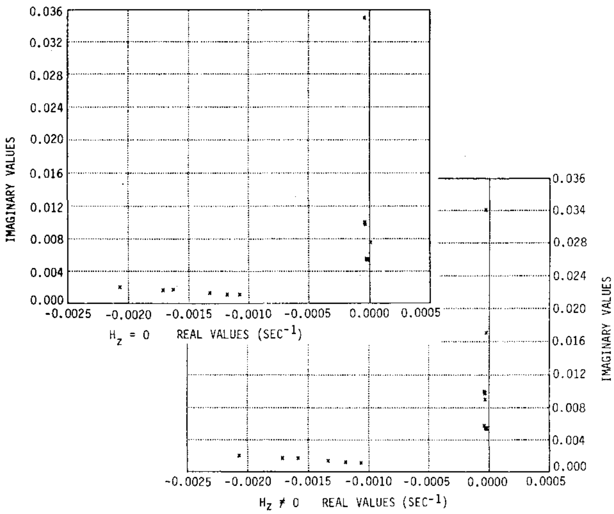

Fig. 3 Mode residualization and output feedback, 10 boom space structure.

Patel obtains H_z (if it exists) by partitioning C and F' in a compatible manner,

$$C = [C_I C_2], \qquad F' = [F_I F_2] \tag{7}$$

so that C_I is square and full rank. Then

$$H_z = F_I C_I^{-1} \tag{8}$$

An extension to this technique, which has made it useful for LFSS design, is in prescribing a method for performing the partitioning in Eq. (7). The system of Eq. (6) is transformed to modal form, and the modal states are reordered so that those involving unstable modes are included in C_I. Then F' is designed by LQG methods, and H_z is calculated from Eq. (8). The stability of the closed-loop system is then assessed, and iteration can be carried out on the choice of C_I and F' to improve performance.

Example Applications

The method described above has been applied to two structural models. The first is a 12-member tetrahedron structure proposed by the Draper Laboratory and used in several studies (see Fig. 1a).[6-8] It sits on a rigid base (with legs of controllable length) and has twelve vibrational modes. Details of the configuration and design are given in Ref. 9. In comparison to Ref. 8, a 4-mode compensator was designed by mode residualization which produced instability; then the output feedback technique was used to stabilize the system.

Figure 2 presents s-plane plots of the 16 modes of Eq. (6) for the system, showing the stabilizing effect of H_z from Eq. (8). Note, however, the lightly damped mode at about 4.7 rad/s. Attempts to improve its damping were not successful.

The second model was taken from Ref. 10, where it is described in detail. It consists of a central disk-shaped hub and 10 booms (five 700 ft long and five 1000 ft long, with tip weights) confined to a plane (see Fig. 1b). There are 14 collocated sensors and actuators mounted on the structure. Six sensors and actuators are located on the central hub to measure and provide actuation of the six degrees of freedom. Eight sensors and force actuators are located at the 40% point on four booms (see Fig. 1b) to measure and effect the flexible modes. Only in-plane motions were considered, including three rigid-body modes and 10 vibrational modes, which reduced the model to 13 modes with seven actuators and seven sensors. A 3-mode compensator was designed by mode residualization to control the rigid-body motion, and its bandwidth was broadened until an instability was induced using mode residualization only. Then output feedback was used to restabilize the system. Figure 3 presents s-plane plots of the system modes for these conditions. As in Fig. 2, light damping is seen for certain modes, and attempts at performance improvement were not successful. Even so, success at stabilizing these examples suggests that constant-gain compensators for a range of simple LFSS structures may be achievable with this technique.

References

[1] Armstrong, E.S., *ORACLS—A Design System for Multi-Variable Control,* Dekker Publishing, 1980; also, NASA TP-1106, April 1978.

[2] Balas, M., "Feedback Control of Flexible Systems," *IEEE Transactions on Automatic Control,* Vol. AC-23, 1978, p. 673.

[3] Gangsaas, D. and Ly, U., "Application of a Modified Linear Quadratic Gaussian Design to Active Control of Transport Aircraft," AIAA Paper 79-1746, Aug. 1979.

[4] Gangsaas, D., Ly, U., and Norman, D., "Practical Gust Load Alleviation and Flutter Suppression Control Laws Based on LQG Methodology," AIAA Paper 81-0021, Jan. 1981.

[5] Patel, R., "On Output Feedback Pole Assignability," *International Journal of Control,* Vol. 20, No. 6, 1974, p. 955.

[6] Strunce, R.R., Hegg, D.R., Lin, J.G., Henderson, T.C., "ACOSS Four Theory," Draper Lab. Rept. R-1338, Vol. 2, April 1980.

[7] Gupta, N.K., "Frequency-Shaped Cost Functionals: Extension of Linear Quadratic-Gaussian Design Methods," *Journal of Guidance and Control,* Vol. 3, No. 6, Nov.-Dec. 1980, p. 529.

[8] Sesak, J.R., "ACOSS One, Phase I," Rome Air Development Center Report RADC-TR-80-79, March 1980.

[9] Bossi, J. and Price, G., "A LFSS Controller Design Tool...," AIAA Paper 82-1566, Aug. 1982.

[10] Wu, Y., Rice, R., and Juang, J., "Control of Large Flexible Space Structures Using Pole Placement Design Techniques," *Journal of Guidance and Control,* Vol. 4, May-June 1981, p. 298.